A Quantum Quench of the Sachdev-Ye-Kitaev Model

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Chaos, Topology, and Dualities in Condensed Matter Theory UIUC

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Quantum matter without quasiparticles

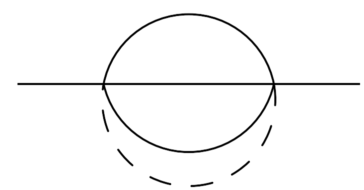
- Want to study properties of systems without quasiparticles
- First: what is a quasiparticle?
  - Long lived additive excitation with same quantum numbers as free particle
- How do we identify systems without quasiparticles?
  - Fastest relaxation $\tau_{eq} \geq \mathcal{C}\frac{\hbar}{k_B T}, \;\; T \to 0$
  - No long lived excitations in *any* basis
  - “Too fast”: cannot study long time behavior with conventional techniques

The SYK model: a solvable system without quasiparticles

- Model of N flavors of Majorana fermions with infinite range q-body interactions

$$H = (i)^{\frac{q}{2}} \sum_{1 \le i_1 < i_2 < \ldots < i_q \le N} j_{i_1 i_2 \ldots i_q} \psi_{i_1} \psi_{i_2} \ldots \psi_{i_q} \qquad \langle j^2_{i_1 \ldots i_q} \rangle = \frac{J^2 (q-1)!}{N^{q-1}}.$$

- Solvable in large N limit
- Maximally chaotic
- Disorder average→melon diagrams, only keep one



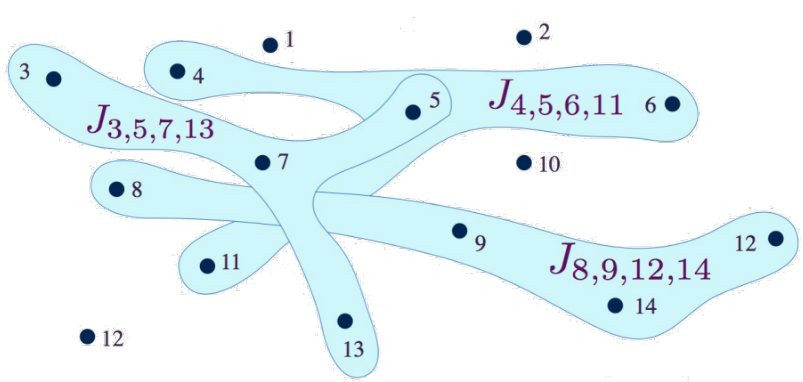
- Full propagator

Strong coupling behavior

- For $\beta J \gg 1$ →"Emergent reparameterization symmetry

$$iG^R(t) = \mathcal{C}(J,\Delta)\left(\frac{1}{\beta \sinh \frac{\pi t}{\beta}}\right)^{2\Delta}\theta(t) \qquad \Delta = \frac{1}{q}$$

- Spontaneously and explicitly broken→ Schwarzian action
- SYK is "dual" to nearly AdS_2



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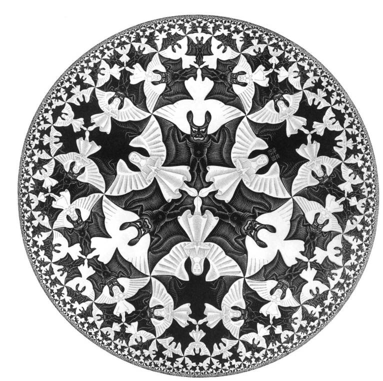
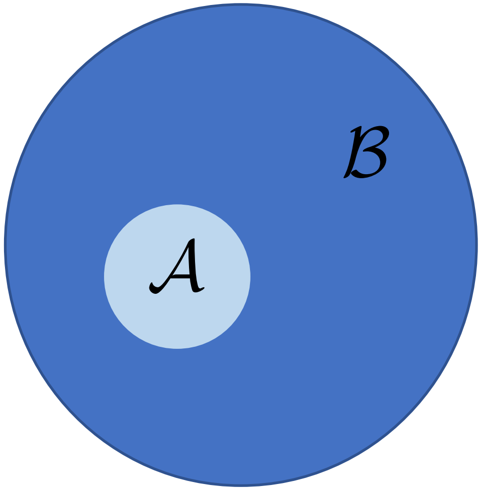


Escher "Heaven and hell"

Quantum quenches and thermalization

- Quench of SYK→ probe nonequilibrium dynamics without quasiparticles
- Possibly reach thermal state
- What is a thermal state?
    - System is its own heat bath for subsystems
    - Steady state, observables reach thermal values
- For SYK, our working definition of thermalization:

2-point function obeys KMS, T from energy conservation

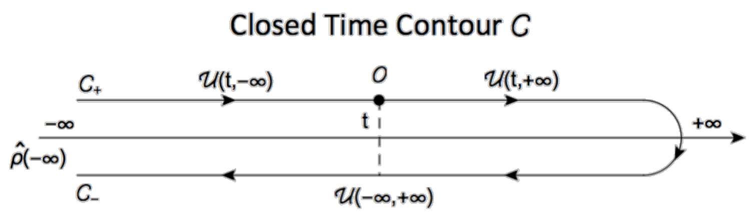


Quench procedure

- Start with SYK model with q and pq interactions
- Turn off pq term instantaneously
- Track evolution of Green's function
  - Does SYK Greens function thermalize?
  - How long does it take (scaling dependence on T)?
- What is the best way to do this?

Green’s Functions on the Closed-Time-Contour

- Out of equilibrium, must study full evolution along contour



- Two Greens functions: $G^{>}(t_1,t_2) \equiv G(t_1^-,t_2^+)$ and $G^{<}(t_1,t_2) \equiv G(t_1^+,t_2^-)$

- Use to form 2 by 2 matrix

- Dyson (matrix) equation from disorder average:

$$G_0^{-1}(t_1,t_2) - G^{-1}(t_1,t_2) = \Sigma(t_1,t_2)$$

$$\Sigma(t_1,t_2) = \sum_i i^{q_i} J_{q_i}^2 G(t_1,t_2)^{q_i-1}$$

The Kadanoff-Baym equations

- How do we study 2-pt function with no time translation?
- Kadanoff-Baym equation directly from Dyson equation
- Go to real time plane get two integro-differential equations:

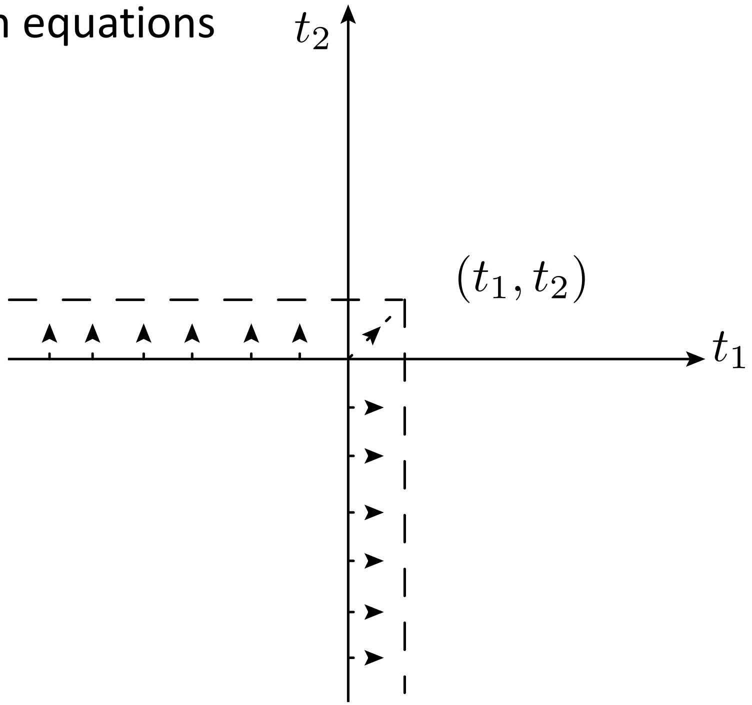
$$G^{>} \otimes G_0^{-1} = \left(G^R \otimes \Sigma^{>} + G^{>} \otimes \Sigma^A\right)$$

$$G_0^{-1} \otimes G^{>} = \left(\Sigma^R \otimes G^{>} + \Sigma^{>} \otimes G^A\right)$$

- For Majorana fermions have condition: $G^{>}(t_1, t_2) = -G^{<}(t_2, t_1)$
- Always true→everything from $G^{>}(t_1, t_2)$!

Causal Structure

- Evolution from integral structure of Kadanoff Baym equations
- Rewrite everything in terms of $G^{>}(t_1, t_2)$
- Step functions→limits of integration
- For point (t_1, t_2) → integrate "rectangle region"
- Pre-quench $t_1 \leq 0, t_2 \leq 0$
- Post quench $t_1 \geq 0, t_2 \geq 0$
- Pass through other quadrants→causal effect



Numerics model

- Consider the SYK+random matrix model (p=1/2 q=4)

$$H(t) = i\sum_{i<j} j_{2,ij} f(t)\psi_i\psi_j - \sum_{i<j<k<l} j_{4,ijkl} g(t)\psi_i\psi_j\psi_k\psi_l$$

  - Only pq: integrable
  - Both terms: “Fermi liquid” $\tau_{eq}^{-1} \sim T^2$
  - Only q: “strange metal” $\tau_{eq}^{-1} \sim T$
- Use $f(t)$ and $g(t)$ to specify quench
- Solve full Kadanoff-Baym equations numerically
- Use Majorana condition → solve for $G^{>}(t_1, t_2)$

Procedure

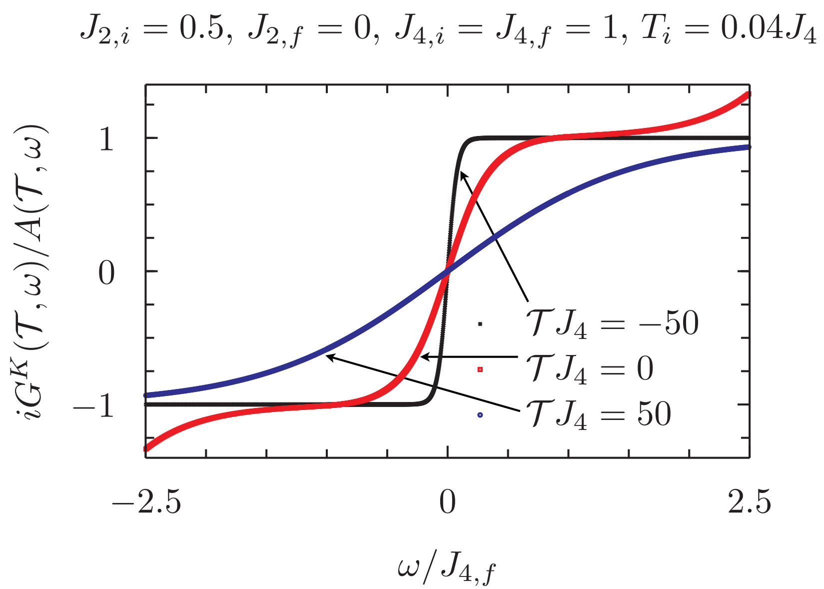
- Solve Dyson equation self-consistently for thermal initial state
- Use as BC for quench
- Immediately post quench, no time translation invariance, define
  - Absolute time: $\mathcal{T} = \frac{t_1 + t_2}{2}$
  - Relative time: $t = t_1 - t_2$
- Near equilibrium, varies slowly with $\mathcal{T}$ look at low frequency behavior
- Wigner transform $f(t_1, t_2) \to f(\mathcal{T}, \omega)$
- Also define $G^K(t_1, t_2) = G^>(t_1, t_2) + G^<(t_1, t_2)$

Thermal state from the KMS condition

- "Thermal" 2-pt function obeys KMS
- KMS→FDT

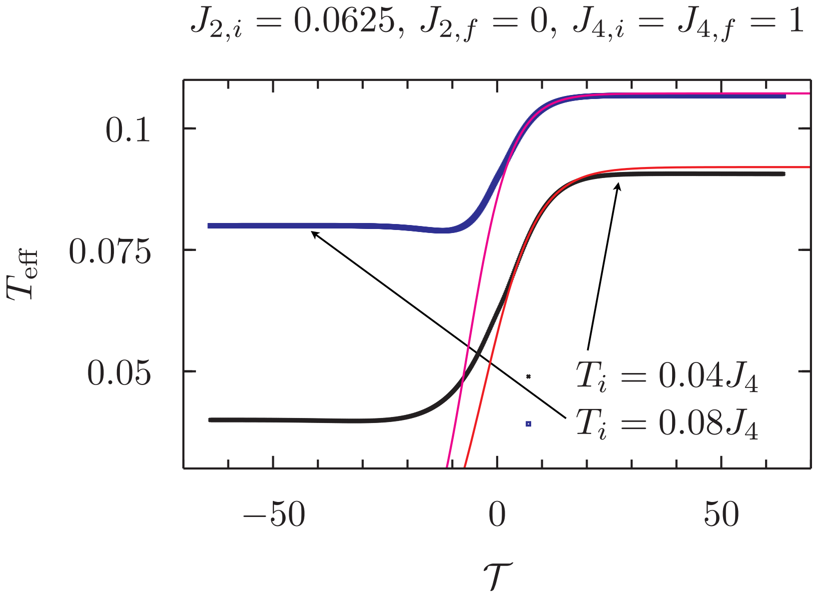
$$\frac{iG^K(\mathcal{T},\omega)}{A(\mathcal{T},\omega)} = \tanh\left(\frac{\beta(\mathcal{T})\omega}{2}\right)$$

- "Effective inverse T": $\beta(\mathcal{T})$
- Start with thermal state
- Right after quench out of equilibrium
- $\mathcal{T}\to\infty$, $\beta(\mathcal{T})$ varies slowly→"thermal"



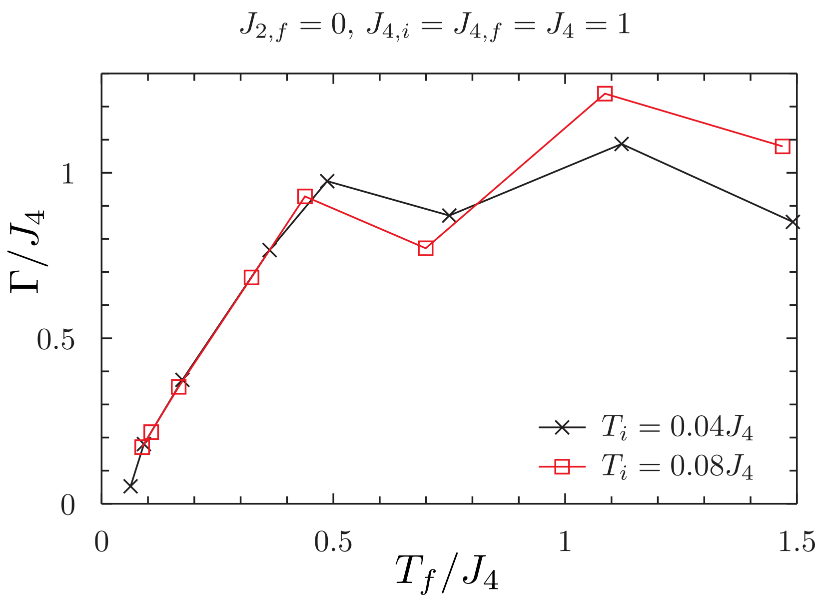
Effective temperature

- T_{eff} from fit of FDT relation
- Relaxes exponentially
- Check $\langle H \rangle = E_f$ throughout quench
- Determines T_{eff}
- Depends on J_2 only through E_f



Relaxation rate

- Know final temperature from energy conservation
- How long does it take to reach final temperature for $\beta J \gg 1$?
- Exponential rate $\Gamma \propto T$
- Higher temperature, Γ controlled by J_4



The large q limit

- Consider large q interaction (after Large N)
- Expand $G^{>}(t_1, t_2)$

$$G^{>}(t_1, t_2) = -i \langle \psi(t_1)\psi(t_2) \rangle = -\frac{i}{2}\left[1 + \frac{1}{q} g(t_1, t_2) + \dots\right]$$

- $q \to \infty$ exponential form of self energy
- Derivatives of KB eqns → Lorentzian-Liouville eqn

$$\frac{\partial^2}{\partial t_1 \partial t_2} g(t_1, t_2) = 2\mathcal{J}(t_1)\mathcal{J}(t_2)e^{g(t_1,t_2)} + 2\mathcal{J}_p(t_1)\mathcal{J}_p(t_2)e^{pg(t_1,t_2)}$$

$$\mathcal{J}^2(t) = qJ^2(t)2^{1-q} \quad , \quad \mathcal{J}_p^2(t) = qJ_p^2(t)2^{1-pq}$$

- Exact solution for p=1/2, or 2

Quench Regions

- General solution in all regions

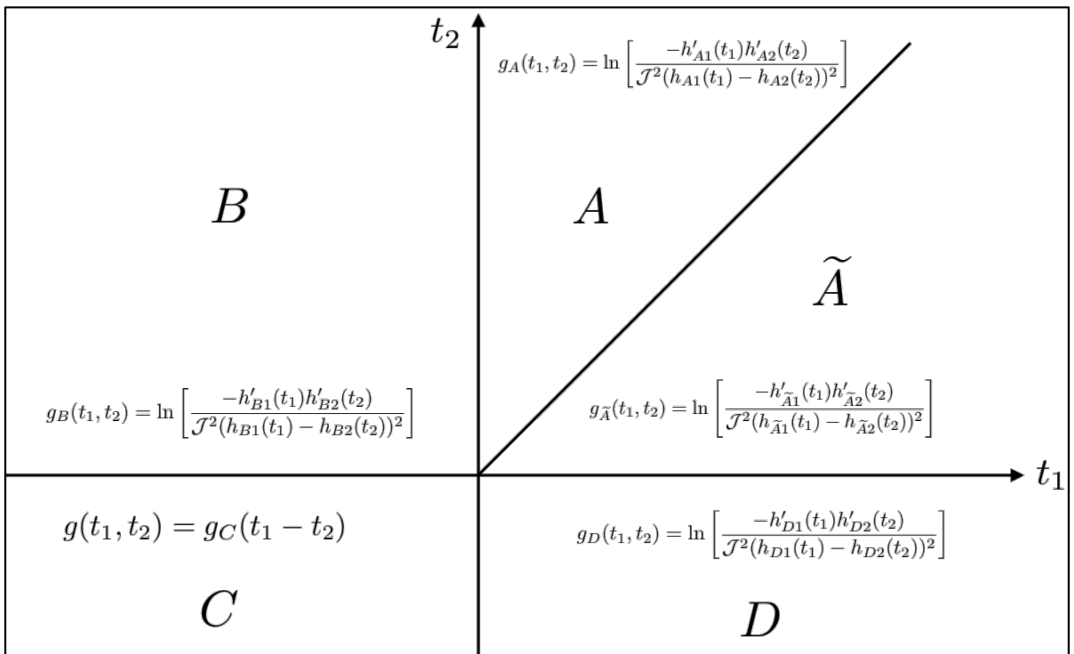
$$g(t_1,t_2) = \ln\left[\frac{-h_1'(t_1)h_2'(t_2)}{\mathcal{J}^2(h_1(t_1)-h_2(t_2))^2}\right]$$

- Majorana condition → $g(t,t)=0$
- For $t_1 \leq 0, t_2 \leq 0$ equilibrium solution $g(t_2,t_1) = [g(t_1,t_2)]^*$
- Structure of integrals in KB equations show this is always true
- **Need to solve in 5 regions**

Quench time plane

$$\frac{\partial}{\partial t_1} g(t_1,t_2) = 2\int_{-\infty}^{t_2} \mathrm{d}t_3\, \mathcal{J}(t_1)\mathcal{J}(t_3) e^{g(t_1,t_3)} - \int_{-\infty}^{t_1} \mathrm{d}t_3\, \mathcal{J}(t_1)\mathcal{J}(t_3)\left[e^{g(t_1,t_3)} + e^{g(t_3,t_1)}\right]$$
$$+2\int_{-\infty}^{t_2} \mathrm{d}t_3 \mathcal{J}_p(t_1)\mathcal{J}_p(t_3) e^{pg(t_1,t_3)} - \int_{-\infty}^{t_1} \mathrm{d}t_3 \mathcal{J}_p(t_1)\mathcal{J}_p(t_3)\left[e^{pg(t_1,t_3)} + e^{pg(t_3,t_1)}\right]$$

$$\frac{\partial}{\partial t_2} g(t_1,t_2) = 2\int_{-\infty}^{t_1} \mathrm{d}t_3\, \mathcal{J}(t_3)\mathcal{J}(t_2) e^{g(t_3,t_2)} - \int_{-\infty}^{t_2} \mathrm{d}t_3\, \mathcal{J}(t_3)\mathcal{J}(t_2)\left[e^{g(t_3,t_2)} + e^{g(t_2,t_3)}\right]$$
$$+2\int_{-\infty}^{t_1} \mathrm{d}t_3 \mathcal{J}_p(t_3)\mathcal{J}_p(t_2) e^{pg(t_3,t_2)} - \int_{-\infty}^{t_2} \mathrm{d}t_3 \mathcal{J}_p(t_3)\mathcal{J}_p(t_2)\left[e^{pg(t_3,t_2)} + e^{pg(t_2,t_3)}\right]$$



Boundary Conditions

- Need 3 BCs
- Get $h_{A1}(0)$, $h_{A2}(0)$ and $h'_{A1}(0)$ from $h_{B1}(-\infty)$, $h_{B2}(0)$, $h'_{B2}(0)$, $g_C(t)$
- Too many BCs
- SL(2,C) invariance: $h(t) \to \frac{a\,h(t)+b}{c\,h(t)+d}$
- Constraint: $ad - bc = 1$
- Result independent of choices for $h_{B1}(-\infty)$, $h_{B2}(0)$, $h'_{B2}(0)$
- SL(2,R) invariance→ "gauge" choice!

Post-Quench Solution

- Choose ansatz $h_{A1}(t) = \frac{ae^{\sigma t} + c}{ce^{\sigma t} + d}$, $h_{A2}(t) = \frac{ae^{-2i\theta}e^{\sigma t} + b}{ce^{-2i\theta}e^{\sigma t} + d}$

$$\sigma = 2\mathcal{J}\sin\theta \qquad e^{-4i\theta} = \frac{(b - dh_{B1}(-\infty))(a^* - c^* h^*_{B1}(-\infty))}{(b^* - d^* h^*_{B1}(-\infty))(a - ch_{B1}(-\infty))}$$

- Find solution for p=1/2

$$g_A(t_1, t_2) = \ln\left[\frac{-\sigma^2}{4\mathcal{J}^2 \sinh^2(\sigma(t_1 - t_2)/2 + i\theta)}\right]$$

- From KMS: $\beta_f = \frac{2(\pi - 2\theta)}{\sigma}$
- **Only depends on relative time: instant thermalization!**

Relation to the Schwarzian Action

- SYK described by Schwarzian for $\beta J \gg 1$

$$\mathcal{L}[h(t)] = \frac{h'''(t)}{h'(t)} - \frac{3}{2}\left(\frac{h''(t)}{h'(t)}\right)^2$$

- Take $h(t)$ from KB equations
- $h(t)$ is solution to Schwarzian EOM

$$\left[h'(t)\right]^2 h''''(t) + 3\left[h''(t)\right]^3 - 4h'(t)h''(t)h'''(t) = 0$$

- Thermalization connected to reparameterization modes
- **Schwarzian should also exhibit instantaneous thermalization**

Final Remarks

- Low energy limit, rate linear in T as expected
- Also depends on q, what do $1/q^2$ corrections look like?
- 2-point function instantly thermalizes, but other quantities do not
  - Which quantities thermalize and on what timescale (some do not)?
  - When does the large N limit break down?
  - What other consequences does this have in gravity?